$$F_r = \frac{U^2}{lg}$$

it represent the ratios of the inertia force to the gravitational forces.

Euler's Number (Pressure coefficient) :-

$$\frac{1}{C_p} = \frac{1}{E_u} = \frac{\text{Inertial Force}}{\text{Pressure Force}}$$

$$= \frac{\text{Mass} \times \text{Acceleration}}{\text{Pressure} \times \text{cross-sectional area}}$$

We know

$$\frac{\rho_2}{\rho_1}\frac{(dq_2/dt_2)}{(dq_1/dt_1)} = \frac{\nabla_2 P_2}{\nabla_1 P_1}$$

$$\Rightarrow \frac{\nabla_2 P_2}{\rho_2 (dq_2/dt_2)} = \frac{\nabla_1 P_1}{\rho_1 (dq_1/dt_1)}$$

$$\Rightarrow \frac{P_2/L_2}{\rho_2 U_2/L_2/U_2} = \frac{P_1/L_1}{\rho_1 U_1/L_1/U_1}$$

$$\Rightarrow \frac{P_2}{\rho_2 U_2^2} = \frac{P_1}{\rho_1 U_1^2} = E_u$$

thus in flows where inerta and Pressure forces Predominate, the pressure coefficient must be the same for dynamic similarity to exist.

—x—

Mach Number :- The Perfect gas law and the velocity of sound is given by

$$P = \rho RT \Rightarrow \frac{\gamma P}{\rho} = \gamma RT = a^2$$

Where a is the speed of sound

Imp. ①

11.0 Laminar flow between Parallel Plates:-

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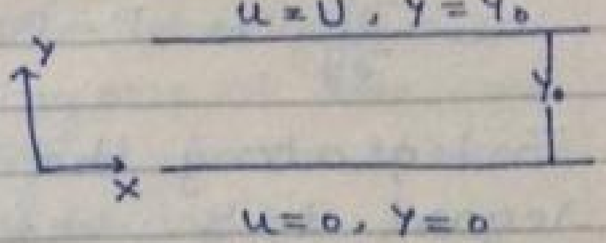

Consider two-dimensional laminar flow of an incompressible fluid of constant viscosity between two Parrallel plates at a distance y_0. the system does not regard to z-axis.

the Equation of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{—— (1)}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \text{—— (2)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \text{—— (3)}$$

the flow between these plates is taken tobe in the x- direction.

then $v = 0$, ~~or~~ $w = 0$ from the continu Equation, we have

$$\frac{\partial u}{\partial x} = 0 \Rightarrow u = u(y, t)$$

Hence the velocity depend on y and t the steady state u depends on y only

$$u = u(y),\quad v = 0 = w \quad \text{—— (4)}$$

From ② And ③ we have

$$0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \quad \text{—— (5)}$$

$$0 = -\frac{\partial p}{\partial y}$$

From ⑥ we have

$$0 = -\frac{\partial p}{\partial y} \Rightarrow P = P(x)$$

Integrating the Equation 5) two times with regard to y, we have

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$$

$$\Rightarrow \frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx}y + A$$

$$\Rightarrow u = \frac{1}{2\mu}\frac{dP}{dx}y^2 + Ay + B \quad —⑦$$

Where A and B are integration constant

Case I :- Plane Couette flow :- the plane Couette flow or simple shear flow between two parallel plates. the upper plate is moving in x-direction with a uniform velocity U. At the Lower plate

$$Y = 0, u = 0 \Rightarrow B = 0$$

At the upper plate $Y = Y_0$, $u = U$

We know $\mu \frac{d^2u}{dy^2} = 0 \Rightarrow u(y) = Ay + B$

$$U = \frac{1}{2\mu}\frac{dP}{dx}.0 + AY_0 + 0$$

$$AY_0 = U$$

$$A = \frac{U}{Y_0}$$

$$\text{i.e. } \frac{dP}{dx} = 0$$

We have $\boxed{u = \frac{UY}{Y_0}}$

It follows that the velocity profile induced in a fluid by moving one of the boundaries at constant velocity is linear across the gap between two boundaries.

Case II :- Generalized Plane Couette flow :-

In this case either of two surface is moving at constant velocity, and there is also an external Presure gradient

$$\frac{dp}{dx} \neq 0$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + Ay + B$$

$$y = y_0$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y_0^2 + Ay_0 + B$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y_0^2 + Ay_0 \qquad [B=0]$$

$$A = -\left\{\frac{U}{y_0} + \frac{1}{2\mu}\frac{dp}{dx}y_0\right\}$$

$$u(y) = \frac{Uy}{y_0} - \frac{y_0^2}{2\mu}\frac{dp}{dx}\frac{y}{y_0}\left(1-\frac{y}{y_0}\right)$$

$$\Rightarrow \frac{u(y)}{U} = \frac{y}{y_0} + \left(-\frac{y_0^2}{2\mu U}\frac{dp}{dx}\right)\frac{y}{y_0}\left(1-\frac{y}{y_0}\right)$$

$$\boxed{\frac{u(y)}{U} = \frac{y}{y_0} + P\frac{y}{y_0}\left(1-\frac{y}{y_0}\right)}$$

Where $P = -\frac{y_0^2}{2\mu U}\frac{dP}{dx}$.

P is a dimensionless Pressure Parameter.

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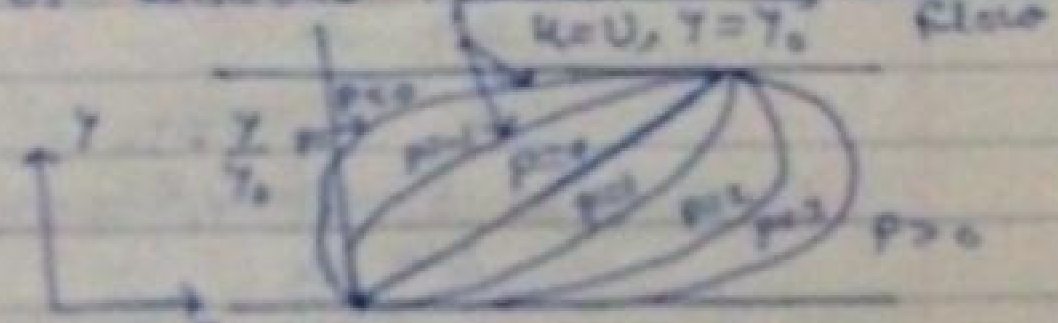
ii) $P > 0$ then $\frac{dp}{dx} < 0$ in the direction of flow

the velocity is +ve between the plates the pressure gradient will assist the viscosity induced motion to overcome the shear force at the lower plate.

iii) $P < 0$ then $\frac{dp}{dx} > 0$ the pressure gradient is increasing in the direction of flow.

(iii) $P = 0$ the fluid motion in the +ve x-direction is entirely due to the action of viscous forces.



the average velocity distribution V

$$V = \frac{1}{y_0}\int_0^{y_0} u\,dy = U\int_0^{y_0}\left[\frac{y}{y_0} + P\frac{y}{y_0}\left(1-\frac{y}{y_0}\right)\right]dy$$

let $z = \frac{y}{y_0}$ then $dz = \frac{1}{y_0}dy$

$$V = U\int_0^1 [z + P(z - z^2)]\,dz$$

$$= \frac{U}{y_0}\int_0^1 [z + Pz(1-z)y_0]\,dz$$

$$= U\left[\frac{1}{2} + P\left(\frac{1}{2} - \frac{1}{3}\right)\right]$$

$$= U\left[\frac{1}{2} + \frac{P}{6}\right]$$